MatheMAGIC

25 Tricks For Teachers

A Manual of Minor Miracles for Magically-Minded Mathematicians!

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Mathemagic: 25 Tricks For Teachers

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Further Reading
1. Books by Martin Gardner: Mathematics Magic & Mystery, Mathematical Magic Show, Encyclopaedia of Impromptu Magic etc. (Penguin)
2. Mathemagic by Royal Vale Heath (the classic!) (Dover)
3. Mathematical Magic by William Simon (Dover)
4. Arithmetricks series by Edward Julius (Wiley)
5. Mathematics Galore! by Budd and Sangwin (Oxford)
6. Magic Courses by Mark Wilson, Tarbell, Bill Tarr, Paul Daniels etc.
7. Memory Books by Harry Lorayne, Dominic O'Brien, David Berglas, Tony Buzan, Alan Baddeley
8. Self-Working Magic Series by Karl Fulves esp. Self-Working Card Magic (two volumes) and Self-Working Number Magic (Dover)

Preface

The world of magic seems distant from the world of mathematics. While magic takes as its premise the need to confuse, baffle and bewilder, but above all to entertain the audience with surprise and mystery, mathematics should surely be seeking to explain, to reassure, to enlighten and to empower, with such clarity that it cuts through the fog of misunderstanding. So where do the two meet?

For many, the world of mathematics is deeply mysterious, and results appear as magically as might the clichéd rabbit from the hat. Surely magical effects would only cloud their view still further? As it turns out, magic tricks have a strong attraction to pupils, especially when they are taught how to perform them, because the knowledge gives them the power to impress their peers, family and even their teachers. They learn how be to cool at school and winners in the dinner queue.

What of the teaching value? Mathematics can be presented as a dry collection of rules and exercises (surely not!) or as a window through which can be seen explanations to many of the world's mysteries. A magic trick provides the interest, and its explanation the demonstration of the power of mathematics to provide answers. Suddenly all that previous work on simplifying algebraic expressions comes into action when explaining why the Number You Thought Of had to be seven.

I was doing magic long before I got serious with the maths, probably not the most common order of things! My father gave me the Ladybird Book Of Tricks And Magic while I was recovering from an illness aged six, and I was doing my first magic show for my sister's birthday party when I was eight. I discovered Martin Gardner books at thirteen, and the Maths connection was made. The magic developed over the years, with more magic shows to boost my student funds, and naturally I managed to make it form part of my PGCE assignments at "teacher school". Not long into my teaching career I became good friends with fellow Maths teacher Andrew Jeffrey, President of the Sussex Magic Circle, and he has been the inspiration for much of my development as a Mathemagician. I am much in his debt.

Every day pupils provide the feedback that essentially says "This is helping to make Maths fun for us". When a lesson begins with an algebraic card trick, or features a child's own pencil sharpener apparently crumpling into thin air, or ends when Sir pushes a pencil through his neck, I can be confident that it is reinforcing our departmental motto,

"Maths is fun and I like it!"

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1. Evens & Odds

Effect
I invite two pupils, Nicole and Sean, up to the front and ask them to share 21 counters between them secretly. I then ask them to do a little calculation between them, and on hearing the result I am immediately able to say whether each child has an odd or an even number of counters!

Method
Those of you who are quicker than me will have noticed that 21 is an odd number. That means that whatever the parity (oddness/evenness) of one child's counters, the other child must be the opposite. This reduces the problem to finding out the parity of just one child's counters. Let's make it more specific by doing a calculation which will enable me to know which child holds the odd number.

I ask Nicole to double the number of counters in her hand and add the number of counters that Sean is holding. If the result is EVEN then Nicole must have had the odd number. If the result is ODD then Sean must have had the odd number.

To thunderous applause, Nicole and Sean return to their seats and we discuss why the trick works.

EVEN x EVEN = EVEN
EVEN x ODD = EVEN
ODD x EVEN = EVEN
ODD x ODD = ODD

EVEN + EVEN = EVEN
EVEN + ODD = ODD
ODD + EVEN = ODD
ODD + ODD = EVEN

As a follow up to this miracle, I ask Doris to come up and help. She takes some of the 21 counters but does not tell me. I count the rest and immediately tell her how many are in her hand. She is not impressed.

Persevering, I invite her to take some counters from the Big Bag Of Counters. Then I take some counters. I tell her that if she has an odd number I will make it even, and that if she has an even number, I will make it odd, just by adding all the counters in my hand. With great suspicion, she counts the counters in her hand and announces that it is an even number. Before she has a chance to see what is in my hand, I tip them all into her pile. Doris counts again, and now the number is odd, just as I had promised.

Just as she is going back to her seat, Doris turns round and looks at me with a big grin on her face. "I know how you did that!" she says, and she sits down smiling.

2. Magic Squares, Any Total

Effect

The 30-second “Countdown” theme is played (or similar) while the pupils quickly pass a Teddy round the class. When the music stops, Sameer is left holding Teddy. I ask Sameer to tell me the number of his house. He tells me that it is 46. Immediately I draw on the board the following square:

26	1	12	7
11	8	25	2
5	10	3	28
4	27	6	9

Quickly we add up each row: 46! And each column! The two diagonals as well! The magic total 46 is obtained in every direction!

But then clever old Suraj and Emily have been adding in other ways. They point out that the corners add up to 46 too, and the 4 middle numbers! Before long the class has found that each corner 2x2 square totals 46, as well as the top/bottom half middle four. Later it is noticed that 1 + 12 + 27 + 6 = 11 + 5 + 2 + 28 = 46, and then Lateral Lisa pipes up with 1 + 11 + 28 + 6 = 12 + 2 + 5 + 27 = 46.

Method

In the square above, the four numbers in the twenties are the only numbers which are altered to make the trick work. I only need to learn this square:

N-20	1	12	7
11	8	N-21	2
5	10	3	N-18
4	N-19	6	9

When I first performed this trick in a classroom, it was back in the days of blackboards and chalk. I had used (cunningly, I thought) a pencil outline on the board which I was then planning to write over with the chalk during performance. Unfortunately for me, the graphite in the pencil was reflective enough to catch the sunlight and be perfectly visible to my audience, thus explaining my surprise as they called out the numbers before I had even chalked them in. Andrew Jeffrey has subsequently given me the far more professional and useful tip that this square could be stuck on the barrel of the whiteboard pen. It could even be memorised!

I don’t go on to reveal this trick to my students for two reasons. Firstly it is in the working repertoire of several professional magicians (I first saw it done by Paul Daniels), but secondly and more importantly, the impact of the apparently endless totals is immediately lost. I prefer to leave them with that sense of wonder.

3. Magic Squares (2n+1) x (2n+1)

Effect

Nikit and Sam are invited up to the interactive white board to slide the numbers 1 to 9 into the 3 x 3 grid so that every row and column adds up to 15. The computer tells them their totals so that they can see how they are getting on. Can they do it before 2 minutes is up? With a few hints (cough, cough) from me they soon complete the task with seconds to spare. As they return to their seats I then offer to show the class how to build magic squares of any size (odd by odd) in record time.
By the end of the lesson, Nikit and Sam, along with many others, have drawn out perfect magic squares of many sizes. Nikit even managed 15 by 15!

Method

Place the 1 in the middle of the top row. Then simply carry on writing down the numbers in order according to three simple rules:
1) The next number is placed NORTH-EAST of the one you have just written.
2) If the box you want to write in is full, then write the next number SOUTH instead of North-East, i.e. in the box below the one you have just written.
3) If you go off the page, then just "wrap around" – top comes back in at the bottom, and right comes back in on the left.

Here's my partial completion of a 7 x 7 Magic Square:

			1	10		
		7	9			
	6	8				
5	14					
13	15					4
					3	12
				2	11	

and so on.

My record on the board is a 21 x 21 square, which I began before I had realised that I would be writing 441 numbers!

These days I use this as a taster lesson for Year 6 children visiting Oaks Park for a sample Maths Lesson. We begin by trying to make the total 15 in as many ways as we can, and then agree that 5 has to go in the middle. When someone has found a solution to the 3 x 3 challenge (there are several, such as reflections and rotations), we share it with the others. As a quick extension, I ask the brightest to find me an anti-magic square, where all the totals are different.
We then go on to explore this construction, with me doing a 7 x 7 on the board and asking the pupils to try either 5 x 5 or 9 x 9 for themselves..

4. Best of 9 Cards

Effect

It's Open Evening and the hapless parents are being dragged round the Maths Is Fun Department by their goggle-eyed offspring to play with all the games and puzzles on display. I pick the parent who most clearly would prefer to be at home right now and ask him to look at a pack of 9 cards which I have just dealt out from my shuffled deck. I then ask him to take out his favourite and return the others face down to the table. By now there is a little crowd forming around my desk, so he knows there is no backing out now. Finally I ask him to show his card to a few others before placing it on top of the other face down cards. I give the remainder of the deck a quick shuffle and complete the pack. I thank the parent for his efforts, and promise that he has done all the hard work. The rest of the trick will be done by the cards and some devastatingly devious algebra.

Picking up the pack, I deal out the first card face up, saying "Ten!". On top of that I deal the second card "Nine!" and so on down to "One!". I then place a face down "lid" on that pile with one other card and repeat the process three more times, making 4 piles altogether.

If a face up card appears with the same number as the one I am saying then I stop and move on to the next pile, starting again from "Ten!". "These cards seem to be telling me something" I mutter mysteriously.

When the last pile is complete, I have some cards in my hand. On the table in front of me are some face up cards, let's say they are a 3 and a 5. ("They are a 3 and a 5!") I now add these numbers together and count down to the eighth card in my hand. It is, of course, the parent's card. Pumping his hand vigorously, I thank him for his time, and explain that Maths really does have many surprising uses.

Method

Johnny Ball named this trick as his favourite card trick of all. It is completely self-working, and the underlying algebra is certainly accessible to school children.

When I place the balance of the deck on top of the spectator's pile of nine, it makes his card 44^{th} from the top (with 8 below it). The fancy counting is just doing 4 x 11.

If there are no matches, then the final "lid" is the spectator's card, but this rarely happens. If I stop part way through, then the number on display tells me how many cards are missing from the intended 11.

If there are n cards in the pile then I need (11-n) to complete it.

As I deal the n^{th} card I am saying the number (11-n), and the number (11-n) is on display if I get a match.

After dealing four piles in this way, the cards needed to complete each pile are still in my hand. Adding the face up cards is equivalent to placing them back on their piles, and the final card is therefore the 44^{th} card. I usually ask the spectator to name his card first. "Seven of hearts" he says. "Not this seven of hearts by any chance?" I ask, as I turn over the final card.

Most packs of cards in school have a few cards missing. If this is the case, then just subtract the number of missing cards from the 9 in the introduction.

5. Four Cards From 12

Effect

Walking around the classroom, I offer the pack of cards to 12 random pupils in turn, asking them to choose a card without letting me see it. When I have returned to the front, I ask the 12 people with cards to stand up. Alex is one of the children standing. I ask him to nominate one of the standing pupils to come forward. He chooses Megan. I ask Meera, also standing, to nominate somebody. She chooses Binal. Similarly, Chris nominates Jack and finally Nigel, the class clown, nominates himself!
Megan, Binal, Jack and Nigel come to the front and the other eight are recruited as Magical Pixies and Fairies. There is a protest from the boys, so they are re-cast as Magical Trolls, which makes them much happier. I collect in the unused cards from the Magical Helpers then give the pack to Gemma, one of my Magical Fairies. Gemma decides to use her glitter gel pen as her magical wand, which is fine by me. I now go to the back of the class.
For the first time I now ask the four at the front to hold up their cards. I then ask my Magical Trolls to give them enough cards to make their value up to 10. With knuckles scraping the floor, they eventually give the cards out as follows:

Name	Megan	Binal	Jack	Nigel
Card Held	Four of Spades	Ace of Diamonds	Jack of Clubs	Joker! No, actually the Nine of Clubs
Extra Cards	6	9	0	1

(I explain that Ace = 1 and any Picture Card (J, Q, K) = 10)

The Magical Pixies are now asked to add up the values of the cards held:
4 + 1 + 10 + 9 = 24

From the back of the room I ask the class if they would be surprised if I told them that I knew the colour of the 24th card in Gemma's pack. They raise an eyebrow. "It's black" I tell them. "Yeah, sure!" they reply, suggesting that it might just be a lucky guess.
"OK, then", I continue, "it's a Spade."
Clever young April points out that 1/4 chance is still not remarkable.
"Hmm. Fairy Nuff. The 24th card in Gemma's pack is the 8 of Spades."

Gemma waves her magical gel pen over the pack and counts down to the 24th card. It is, as predicted, the 8 of Spades. I thank my Magical Assistants for making the trick work and they all go back to their distinctly non-magical seats.

Method

Yes, this is simply another version of the previous trick. Just note the bottom card on the deck, and then place the 8 unused cards on the BOTTOM before handing it over to your own Magical Fairy. Once again the predicted card is 9th up from the bottom.

4 cards held + 16 cards given out + 24 total value = 44, as before.

6. Think Of A Number And Variations featuring Jam Jar Algebra

Effect

I ask everyone in the class to write down a number between 1 and 20.
Now double it.
Add fourteen. (Pause and recap)
Now divide your answer by two.
Do I know what your answer is? No? You're right!
Finally take away your original number.
Put your hand up if your answer is seven!

Method

I write the instructions on the board, and Elise assures me that she can find a number which doesn't work. I promise her a commendation if she can find one, but being the softie I am she gets one for trying at the end of the lesson anyway. Jordan and Romaine are impressed, so I ask them why it works. This is a bit tricky, so we try a simpler version:

Think of a number.
Add seven.
Take away your original number. The answer's seven!

Vicky and Michelle are convinced that Sir has lost it big time by now. I offer to make it more complicated:

Think of a number.
Add seven.
Double it.
Divide by 2.
Take away your original number. Even this is too obvious!

I encourage the pupils to make up their own version.
Can they make one that always gives the answer 5?

Jam Jar Algebra

I take two identical jam jars and a pile of Multi-Link cubes for this demonstration.
We work through the original version of TOAN. The class agrees to start with 9.

I put 9 Multi-Link cubes inside one jar so that they can be seen clearly.
When I "double it" I simply produce a second jam jar and put 9 cubes in that too.
Two jars side by side. Fourteen extra cubes are placed on the desk in front of the jars.
Halving it is easy – I just create two piles with a jam jar and seven cubes in each pile.
Finally I take the original jam jar away: 7 cubes left.
We try again with four in the jar, and the result is the same.

"Can you see why it always works?"
Elise, perhaps you would like to explain it to the class!

7. Fibonacci Sums

Effect

Abu and Ali have been learning about the Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, etc. so I invite them to the front. I ask Abu to write down a number under 20, and Ali to write below it another number under 20. They are then asked to create a Fibonacci sequence using these two starting numbers. I ask for ten terms altogether.
This is what we see on the board:

17
4
21
25
46
71
117
188
305
493

I then challenge Abu and Ali to a race,
They are allowed calculators, but I am not. Of course, like fools, they agree.
Add up the ten numbers, I say. Starting from…. Now!

Calmly I write something on my Show-Me board and put it face down on my desk. I then stand in front of the board and make a great show of trying to do all the mental calculation in my head "Carry the two makes fifteen, so that's four hundred and eleven in the tens… etc." A short time later Abu and Ali have blurted out the answer: 1287 and laugh at me shaking my head.
I then ask them to turn over the Show-Me board which they had not noticed on my desk. On it is written:

The total is 1287.
Sorry, I win!
Thanks for taking part. *Mr F*

Method

This is excellent for use when doing algebraic simplification. As soon as I see the seventh term go up on the board, I simply multiply it by 11 (see Trick 13 for method). If the original numbers were a and b respectively, then the subsequent terms are just $a+b$, $a+2b$, $2a+3b$, etc. The seventh term is $5a+8b$ and the sum of the first ten terms is $55a+88b$ as required.

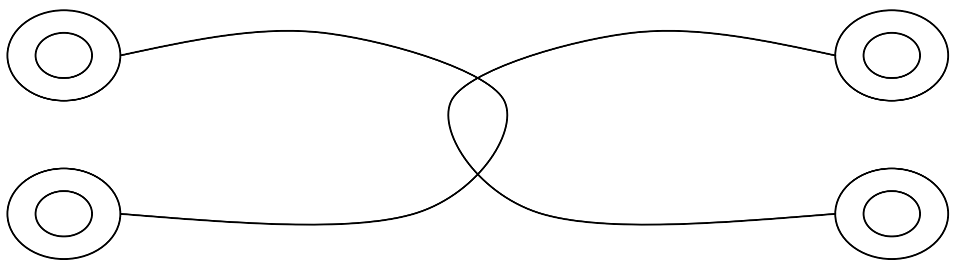
The Show-Me board can be pre-written with everything except the total before you start, and held as part of your files and folders as you introduce the trick. Putting it on the desk then looks quite natural, as if you were simply putting your things down so that you could concentrate on the task. Done while you're explaining the task, you could even have it down before the last number is written and then NOBODY would notice! You could be a legend in your own lunchtime.

8. Tip-Top Topology: Rope Escape & Linking Paperclips

Effect

I explain to the class that topology is a branch of mathematics that is all to do with surfaces, knots and very hard sums. They can calculate, for example, Jonathan, that if you carry on wearing your tie like that you will get three detentions by the end of the week. How do you become a topologist? You simply start as a bottomologist and work your way up…

Mike and Carla have agreed to help this time. I show them two lengths of soft rope and tie a slip knot at both ends of each one. Carla puts one fair hand through each loop in her rope and I gently pull the slip knots tight so that the rope hangs between her wrists. Big Mike does the same, but just before his second hand goes in, I link the two ropes by passing one end through Carla's loop:



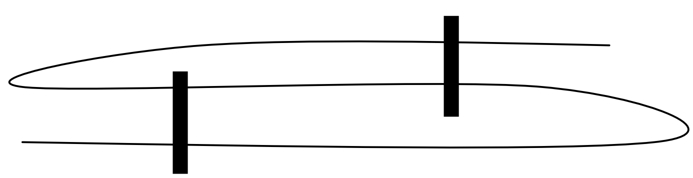
The challenge is now simple – separate yourselves without undoing the ropes!

Method

This usually gets a huge laugh as they clamber over each other with encouragement from the rest of the class. In actual fact, the only way to succeed is for Carla to take a loop of rope from Mike and to pass it up under her wrist band, over all her fingers, then pull it free.

Linking Paperclips

Take a rectangle of paper (a bank note is good for sustaining interest!) and fold it into an S-shape. Attach two paper-clips as shown in this birds-eye view:



When you pull the paper open sharply again, the paper clips fly off and land linked! The paperclips are actually edge-identifiers on the folded paper surface, and since the edges which have been brought together now overlap, that overlap is transferred to the paperclips when the surface is unfolded again. This is a little topological puzzler more than a trick, but the pupils are encouraged when it finally works for them too. There seems to be a "knack" for this, which soon spreads around the class.

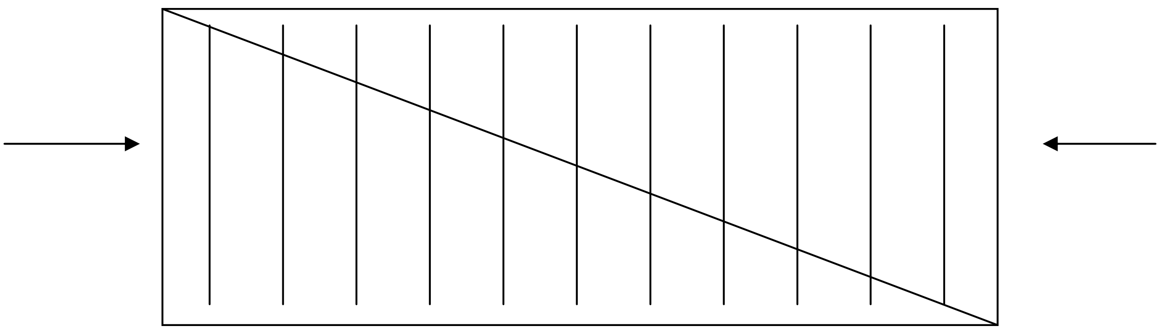
9. Vanishing Line and Vanishing Area

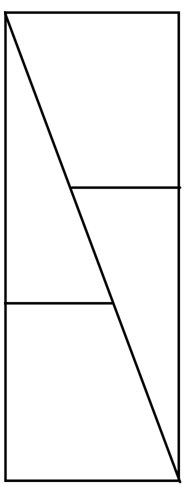
Effect

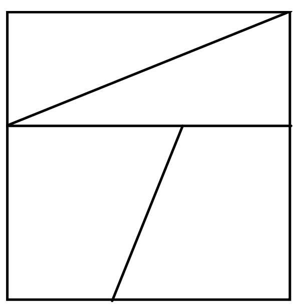
Kirsty-Ann wants to help now. Everyone knows she's one of the brightest girls in her class, if not the year. I ask her if she can count to eleven. She gives me her "look". Smiling sweetly, I ask her to count the number of lines in my picture on the board. With hardly a pause, she confirms that there are eleven lines. I ask her if there will still be 11 lines if I move the pieces in the picture. "Obviously!" she replies, but then is momentarily baffled as those 11 lines become 10 right before her eyes.
I then change the picture to show a rectangle made of four pieces. She tells me the area is 65cm^2 before I even ask. But then I move the pieces around to make a new rectangle. "…but that is now only 64cm^2! What's going on?"

Method

In the first instance these two triangles are simply pushed closer together.







The triangle is right angled with Base 3 and Height 8. There are two of these.

The trapezium has parallel lengths of 3 and 5 with a height of 5. There are two of these.

The first area is 5 x 13 = 65, and the second area is 8 x 8 = 64.

There is a third arrangement which looks like an area of 63. Can you find it?

I like to preface this with Crazy Finger Counting (see Finger Magic).
The true area is obviously 64. The first fails (thanks to the two different gradients on the two shapes) because of the very thin parallelogram running down the diagonal. It has an area of 1 as might be expected.

10. Magic Age Cards

Effect
When you are trying to guess someone's age, it is quite impressive when you guess correctly for a complete stranger. When it's a Year 9 class you have been teaching for a whole year, you don't gain much credibility for guessing that a pupil is 14. Undaunted, I ask Nimrita to come up and help (don't forget to bring your magic wand). Nimrita is sorry that she didn't bring it to school today. We think we can cope. I ask Nimrita to think carefully about the day of the month on which she was born (e.g. 5 if she was born on the 5th etc.). I then show her a set of cards, which appear onscreen via PowerPoint, simply asking her to say whether or not she can see her number on the card each time. After the last card has flashed up I tell Nimrita that her birthdate was the 27th. She is as close to being amazed as her street cred will let her.

Method
We look at the cards Nimrita picked and try to spot patterns in the way the numbers are set out. Would anyone else like to try? I give them a worksheet each so that they can try this out in pairs. Surely there's a quick way to find the number chosen?

1	3	5	7
9	11	13	15
17	19	21	23
25	27	29	31

2	3	6	7
10	11	14	15
18	19	22	23
26	27	30	31

4	5	6	7
12	13	14	15
20	21	22	23
28	29	30	31

8	9	10	11
12	13	14	15
24	25	26	27
28	29	30	31

16	17	18	19
20	21	22	23
24	25	26	27
28	29	30	31

Nimrita said "Yes!" to all the cards except the one beginning 4, 5, 6, 7.
I then simply added up the first (key) number on each chosen card to tell me her birthdate: 1 + 2 + 8 + 16. This is much easier to do if I just keep adding as I go along.

Pupils can be encouraged to investigate the method by setting up a table with six columns:

Birthdate	Key Card 16	Key Card 8	Key Card 4	Key Card 2	Key Card 1
1	-	-	-	-	YES
2	-	-	-	YES	-
3	-	-	-	YES	YES

and so on. Perhaps they may even "discover" binary arithmetic!

How can the cards be extended to cover all number from 1 to 63 ?

11. The Amazing Number 9 – The Expunged Numeral

Effect

Every pupil in the class writes down their telephone number (without area code) or a number of as many digits. They then shuffle these digits around to make a smaller number. For example, 5249 can be shuffled round to make 2954. The more digits in their number, the better! Now they subtract the small number from the big number and keep the answer to themselves. I recap on the instructions at this point to make sure everyone understands what to do. Now they put a ring around any digit in their answer, "but not zero, because that already looks like a ring" and add up all the OTHER digits:

$$3\;4\;1\;6\;2\;2\;9 \quad \Rightarrow \quad 3+1+6+2+2+9 = 23$$

I now go around the class, asking for the final answers and IMMEDIATELY telling the pupils which number they circled:

"17" "You circled 1"
"24" "You circled 3"
"51" "You circled 3 as well"
"23" "You circled 4" (above)
"11" You circled 7" and so it continues, right round the room, as fast as I can speak.

Method

One of the most incredible properties of our number system is its power to make tricky calculations very easy. Since we write our numbers in base 10, it follows that when we subtract the digits from a number we always end up with a multiple of 9. (This is basically saying that 1000 – 1 and 100 – 1 and 10 – 1 are all multiples of 9.) One of the properties of any multiple of 9 is that its digital root (the sum of its digits, with the addition repeated until a single digit is reached) is also 9.
Putting these two together we can see that the answer to the subtraction is always going to be a multiple of nine. All I have to do is answer back with the smallest number that will make their total up to a multiple of 9. If it is already a multiple of 9, then they must have crossed out 0 or 9, but zero was forbidden, hence 9 is the answer.

This trick was described well over a century ago by "Professor Hoffmann" on p237 of his book "More Magic" (1889) where it is offered as a routine for the Victorian Stage Magician. "Expunged" just means "Crossed out". I have amended this to ringing because it provides a neat way of eliminating the problem faced when either a zero or nine is "expunged".

12. Grey Elephant From Denmark

Effect

This one is best worked with a crowd rather than an individual, so I find myself as the warm-up guy for the School Talent Show. I take the audience through the instructions carefully:

Think of a number, any size.
Multiply it by 10.
Take away your original number.
Add the digits together. For example, if your answer is 16, you do 1 + 6 = 7.
If you still have a two-digit number then add the digits together again.
Think hard about the number in your head.
If it is smaller than five, then add 5 to it.
If it is bigger than five, then take 5 away from it.
If it is equal to five, then leave it alone.
Let's say your number was six. This is bigger than 5 so you do 6 – 5 = 1.
Now imagine a simple code.
A = 1, B = 2, C = 3 and so on. Think of the letter that goes with your number.
Now think of a country beginning with that letter.
Think of the second letter of your country.
Think of an animal beginning with that letter.
Finally think of a colour you would normally associate with that animal.

Well, I don't know what number you were thinking of, but I am surprised to see so many GREY ELEPHANTS in DENMARK!!

Method

The wording above is crucial. It contains many psychological tricks to keep the cynics off the scent. For example, the 2-step multiplication by 9, the impossible examples and red herrings, and the code that says "and so on" instead of D. The final four lines have to be said briskly to force people to think of the first thing in their head.

As already discussed, the digital root will always be 9, so the letter obtained will always be D. When pushed, most people can only think of D – Denmark, E – Elephant and then you are safely home.

Played with a large audience, the collective gasp of surprise is enough to bring spontaneous applause as you walk off stage. With smaller groups, instead of speaking the conclusion, I simply hold up a picture of a grey cartoon elephant marching towards a map and flag of Denmark. Even if they know the trick, this usually gets a laugh anyway.

13. Multiplying By 11

Effect

I write a single digit on the board and tell the pupils that when I start the clock, we are going to race to see who can multiply that number by 11 the fastest. ("Wait for it…!")
Ready …Steady …GO! And of course I am deafened by a chorus of correct answers. before I have even returned to the board.
"OK," I admit, "you win that round. Let's move on to Level 2"
I write down the number 3143221609 and shout GO!!
To a wall of deafening silence I quietly write down the answer 34575437699.
"Did anyone beat me?", I ask gently as I turn round to face the sea of open mouths.
By the end of the lesson everyone in the room is able to repeat my stunt.

Method

Multiplying by 11 stunts are all based on the fact that 11 = 10 + 1. In other words, you multiply by 10 first (shunting all the digits left) and then add on the original number.

7 => 70 + 7 = 77 etc. "double the digit" as children often say.

Two-digit numbers are more interesting: Put the sum of the digits in between the two digits!
43 x 11 = 473 51 x 11 = 561 and so on.
Why? It comes down to 430+043 and 510+051 – the sum ends up in the tens column.
When the sum is more than 9, just carry the 1:
57 x 11 = 627 94 x 11 = 1034 and so on.

With a long string of digits you are simply generalising the two-digit pattern.
WORK FROM RIGHT TO LEFT SO THAT YOU INCLUDE THE CARRIES.

In my example, I write down the units digit, 9.
The I add 0 + 9 = 9 which goes on the left of the first one.
6 + 0 = 6, put this in front of the 9 you have just written.
1 + 6 = 7, put this in front of the 6 and so on.
Finally you write the first digit on its own at the beginning of the number.

This leads neatly on to a handy little divisibility test for 11.
Put a dot above the first digit and a dot below the second digit. Alternate between dot above and dot below all the way through the number.
Add up the digits with a dot over them. Call this total A.
Add up the digits with a dot below them. Call this total B.
Is A – B zero or a multiple of 11? If so, then so too was the original number.

In my example, 3 + 5 + 5 + 3 + 6 + 9 = 31
4 + 7 + 4 + 7 + 9 = 31
31 – 31 = 0 so 34575437699 is genuinely a multiple of 11.

14. 1089

<u>Effect</u>
Rita has been quiet for a while, so I ask her to come forward to the board. I explain that my prediction is already made, and show a sealed envelope which I tape to the side of the board. Rita loves “takeaways and adds” so this one is perfect for her. I ask Rita to write down three different digits and arrange them to make any three digit number. I then ask her to reverse the digits and write down the new number. Rita has 621 and 126 on the board. Confidently I ask Rita to take away the small number from the big number. As I thought, Rita comes up with the correct answer:
621 – 126 = 495.
Now I ask her to reverse this number, but this time to ADD the two together.
Good girl! Rita has worked out 495 + 594 = 1089.
Now I reveal my prediction: “Six thousand, eight hundred…” but I don’t get any further because the rest of the class is laughing.
“Whoops! I had it upside-down: here it is!” and I turn over my board to show 1089.

<u>Method</u>
I like to use this self-working number trick as a surprise finish to Summer School classes with Level 3/4 children like Rita. It reinforces the need for careful subtraction with decomposition. The algebraic proof is best left until a lot later on! Here goes:

Let the larger number be abc i.e. $100a + 10b + c$.
The smaller number is $100c + 10b + a$.
Remember that $a > c$. Proceed either in expanded notation, or in columns as below.

Subtracting:

Hundreds	*Tens*	*Units*
a	b	c
c	b	a
$\mathbf{(a-1)-c}$	$\mathbf{(b-1)+10-b=9}$	$\mathbf{c+10-a}$

Reversing and adding:

Hundreds	*Tens*	*Units*
$(a-1)-c$	$(b-1)+10-b=9$	$c+10-a$
$c+10-a$	9	$(a-1)-c$
9 + 1 = 10	**8**	**9**

Investigate – what would happen if you started with a four-digit number?
A 2-digit number?
Why do the three digits have to be different?

What other questions can you make up? Now try to answer them!

Thanks to Andrew Jeffrey for the 6801 false ending!

15. ABCABC

<u>Effect</u>

"Wouldn't you be amazed", I ask my jaded middle set Year 8 class, "if I could ask you to type any secret number into your calculator, yet still be able to tell you instantly what it was?"

"So would I," I continue, pretending to have heard a response, "but this one is almost as good. Get out your calculators!"

So the wind-up continues: "Type in absolutely any number you like, but it must have three digits. Oh yes, and the digits must all be the same. Oh and one tiny thing - the first digit has to be a two."

Eventually I give in and "get on with it". They each type in a different 3-digit number.

"Now type it in again, so if you typed 123 (I bet YOU typed 123, didn't you, Ryan? And YOU, Vicky! Try another!) then you would get 123123 on your display. Here's mine – 290290. But don't show me yours.

"Now I am going to make some predictions. I bet you that if you divide your number by 7 then you will get a whole number answer. Try it!

"You were a tiny bit impressed by that I can tell. Try not to show it though, because that wouldn't look cool. Just try dividing your answer by 11. That goes exactly as well, doesn't it! Ooh I'm getting so excited.

"Now this one you will love. Divide by 13 and you will see my prediction staring you in the face!"

I take my bow to the invisibly and inaudibly cheering crowd and return to my desk.

<u>Method</u>

You should be able to work this explanation through with the class. It's all about factors and multiples.

The number *abcabc* is a multiple of *abc* since *abc* x 1001 = *abcabc*.

Now, what are the factors of 1001? (Class exercise to find 1001 = 7 x 11 x 13)

So if we start with *abcabc* and divide by each factor in turn we simply reverse the original multiplication and return to the very first number *abc*.

Working with other numbers of digits?

It might be worth an investigation to see if this trick has analogues for 1, 2, 4, 5, 6… digit numbers.

Let's see: 1 digit e.g. 7 x 11 = 77 and 11 is prime. Not very interesting.

2 digits e.g. 43 x 101 = 4343 but 101 is prime again.

Question – are all the others prime??

3 digits we know about: 101 = 7 * 11 * 13

4 digits e.g. 5187 x 10001 = 51875187 and WOW! 10001 = 73 * 137.

I can't see my Year 8s getting very emotional over that one though.

5 digits e.g. 12345 x 100001 = 1234512345 and 100001 = 11 * 9091

Hardly a reputation-maker there either. Better stick with ABCABC after all.

16. Cards That Spell Their Own Names

Effect

Well here we are on another jolly staff INSET day looking at Literacy Across The Curriculum. I say I have something to offer from the Maths Department, that well-known mine of poetic beauty and linguistic perfection. I produce a little packet of cards "and as you can see they are in no particular order" and hold them face down. I deal the top card to the bottom of the pile in my hand and say "A". My colleagues look at me with a mixture of alarm and pity. Undaunted I continue to deal one card at a time to the bottom of the little packet, "C", "E". And the next card I turn over and show to be the Ace. This goes face up on the table. Carrying on, I deal to the bottom: "T, W, O spells …" and indeed the Two is the next face up card to go on the table. I continue in this manner until I have just two cards left, which change places with each letter I pronounce: "Q, U, E, E, N spells …" and that must mean I am left with… and the King goes on top.

Method

It is good to challenge the pupils to work out the required order, perhaps initially with just the Ace, Two and Three (A, 3, 2 is the set up) or Ace to Five (532A4). I usually show them that they can do it by lining up a row of empty boxes or place holders. ACE must go after the third letter, so in the fourth box. TWO goes four boxes after that, wrapping round to the beginning and ignoring the Ace. THREE goes six places later, ignoring the Ace and the Two, and so on.
For Ace to King, the order is: 3, 8, 7, A, Q, 6, 4, 2, J, K, 10, 9, 5 viewed as a face down deck, which certainly looks as if "they are in no particular order".

GOING FURTHER

1) What about doing this trick in other languages?
2) Make a set of cards for every number from 2 up to 13 cards per set.
2) What about making a set of cards for shapes rather than numbers? On the back of the top card I would have a picture of a square. If I did the spelling correctly, S-Q-U-A-R-E, I would turn up a square as my next card. The new face down card would then give me the next shape to spell. Next up would be RHOMBUS (conveniently the same number of letters as DIAMOND!) and so on. Thus we have a little packet of cards which can be used for self-checking spellings in a variety of contexts. How could you get round the problem of needing a different face down card every time?
3) Deal one card to the bottom, one card to the table. One card to the bottom, one card to the table. Keep going all the way through the packet. When you have finished, the cards on the table are Ace to King all in the correct order. What was the order in the original packet?
4) Take a new shuffled pack of 52 cards and spell out every card from Ace to King, keeping them face down on the table as you deal out each letter. As you reach the G of K-I-N-G you are placing the last card on the table! (Thanks to Paul Daniels for that one!)

17. Stacking Dice

Effect

Year 9s, post-SATs and feeling like they know it all, which I'm sure they do since they have had you as their teacher. As a reward for all their hard work you finally agree to let them do some extra algebra. You explain that it involves lots of dice (chorus of "Never say die!" or "Die another day"), a steady hand, and some even steadier adding up. As you are giving it the big build-up you are handing out about six dice to each table. On your way back to your desk, you stop at Barry's desk and pick up one of his dice. "Typical, Barry! Trust you to get the only die that doesn't work!" You show him how the bottom number keeps changing when you turn it over (see Dice Magic) and replace it with "an ordinary one". Barry is of course baffled that he can't see the slightest difference…

You ask each table to make a stack of four dice. Carefully you explain, using pictures on the board if necessary, that there are exactly seven hidden faces, since all the vertical ones can be seen and the hidden ones are where two dice touch or where the bottom die touches the table. You ask each table to add up their hidden faces (starting at the TOP – demonstrate this) and tell you their totals. Clearly there are many different answers in the room. This can be tried again with a stack of five or six dice until they get the idea that the totals are going to be different from one table to the next – an important concept.

Finally you ask each table to make a stack of either three, four, five or six dice and to write down the total of the hidden faces. When they have done that you zoom round the room, shouting out the totals at lightning speed – That one adds up to 38! That one is 23! That one makes 30! And so on, around the room until you have correctly predicted every total.

Method

Your Year 9s might actually like this one, especially when they are able to derive the little algebraic formula that you are using. Since opposite sides of the dice add up to seven, the total of the horizontal faces must be $7n$where n is the number of dice in the stack. But we can see the top number d, so the total of the hidden faces is simply

$$\text{Total} = 7n - d$$

Since all you need is the number of dice and the number of spots on the top, you can "steal" this information, as magicians say, with an almost imperceptible glance at the stack. When you are busy "mind-reading" the total of hidden faces, you can then have your back completely turned against the stack so the stunt looks completely impossible.

Have fun with this one – it's a good one!

18. The Seventeen of Diamonds

Effect

It's Wednesday, last lesson, and my Year 10s are in no mood for quadratic factorisation. Come to think of it, they said they weren't in the mood last lesson either, nor the one before that… With fearless disregard for my own safety I lurch into the classroom and greet their suspicious teenage faces with a fearsome grin. "I've just bought this new card trick!" I cry maniacally, "and you lucky people are going to be my first victims, I mean my first audience!"

I ask the long-suffering Puja to humour me once again, and she does a beautiful twirl and steps up to the board. Big Mike is also invited, and he comes stomping up to take his customary role of Glamorous Assistant. He grunts his assent. Puja is asked to write on the board any number between 500 and 1000 in her beautiful handwriting. Big Mike is asked to move gracefully among the spectators to borrow a calculator, which he does as gracefully as his size 13 flat feet will allow.

I announce to my rapt audience that I have a prediction in my pocket – a playing card which will reveal the final answer. Carefully I raise the top edge from my breast pocket so that they can see a little bit of the back. This card then remains in view.

I then turns my back to the board and asks Puja to write on it (the board not my back!) the answers Big Mike gets at each step of the calculations, so that everyone else apart from me can see them.

After a series of mathematical operations on Puja's number, I dramatically reach into my pocket for the playing card, while at the same time asking Puja for her final answer.

Disaster! She says "Seventeen"!

"Great trick, Sir! Looks like you need a bit more practice! Ha! He messed up!" etc.

Even Big Mike is grinning now, showing both his teeth. Puja smiles sweetly.

But the card comes out of my pocket anyway, and it is in fact the Seventeen of Diamonds. Suddenly quadratic factorisation is enormously appealing.

Method

This is another "Sucker trick" where the spectators are led to believe that the magician has got it wrong. Whoever has heard of the 17 of Diamonds?? Clearly it is a think-of-a-number effect which forces the number 17. Here's my method:

Think of a number	N
Add 25	N + 25
Double it	2N + 50
Add the original number	3N + 50
Add 1	3N + 51
Divide by 3	N + 17
Subtract the original number	17

The 17 of diamonds can either be made by sticking a false front to an ordinary card, and sticking 17 little red diamonds on the face, or simply taping together the short edges of the 2, 4, 5 and 6 of diamonds to form a 4 x longer card and then rolling them up and placing the roll in your pocket. A concertina of four cards is also effective as it can be dropped open while the card back is being turned round.

19. Ten Guests Into Nine Rooms

Effect
The magician tells a story – this trick is entirely aural but nevertheless most baffling. The story concerns ten travellers who each need a room for the night. When they finally arrive at the hotel, however, they are told there are only nine rooms. Nevertheless, the hotel proprietor has a cunning plan that soon has all ten safely tucked up in their own room. Or does he?

Method

Ten weary, footsore travellers,
All in a woeful plight,
Sought shelter at a wayside inn
One dark and stormy night.

"Nine rooms, no more," the landlord said,
"Have I to offer you.
To each of eight a single bed,
But the ninth must serve for two."

A din arose. The troubled host
Could only scratch his head,
For of those tired men no two
Would occupy one bed.

The puzzled host was soon at ease –
He was a clever man –
And so to please his guests devised
This most ingenious plan.

In room marked A two men were placed,
The third was lodged in B,
The fourth to C was then assigned,
The fifth retired to D.

In E the sixth he tucked away,
In F the seventh man,
The eighth and ninth in G and H,
And then to A he ran,

Wherein the host, as I have said,
Had laid two travellers by;
Then taking one – the tenth and last –
He lodged him safe in I.

Nine single rooms – a room for each –
Were made to serve for ten;
And this it is that puzzles me
And many wiser men.

20. Probability Snap (Two Packs)

Effect

Poor old Jessica. She really believes that she is going to outsmart me with her cunning when I offer to play one of my crazy scams. You've arrived just as she has accepted my latest bet, and we're playing a bizarre kind of Snap game, each of us with a pack of cards in our hands which we have shuffled to within an inch of its life.

This was my challenge:

"I bet you that I can control the cards in your hands so that however hard you shuffle them, at least one will be in exactly the same place as it is in my pack! 10p to play, and this delicious Mars Bar if you beat me!"

Jessica thinks that 10p for that Mars Bar is money well spent, especially since she is so sure to win. She steps forward, purposefully yet naïvely confident as usual.

We sit facing each other across the table, with the class gathered round, scrutinising the two of us for any sign of foul play. We deal our first cards out at the same time. Jack and Four. Jessica knew this was going to be easy. Seven and Three. This is laughable. Then a crisis! Eight of Hearts and Eight of Clubs! I assure Jessica that this is NOT Snap, since the suits are different. I was waiting for numbers and suits to be the same. Two identical cards. This cheers Jessica up no end – this really is going to be easier than she thought. We keep dealing, and soon we are in the second half of the pack, and our eyes are moving closer to the two piles on the table. Then it hits, and the room suddenly goes very quiet. There on the table, turned up at the same time, is a Four of Spades from me and a Four of Spades from Jessica.

"I think that makes Snap" I say, nonchalantly pocketing the 10p on the side and picking up the Mars Bar. "Looks like I got lucky again."

Method

Probability specialists will undoubtedly recognise this as a question of "derangements" – the probability that everything will be in the WRONG place (for which Jessica would win). Unfortunately, as the number of items increases, this gets closer and closer to 1/e where e is that ubiquitous number 2.71818…

This puts Jessica's chance of winning at 36.8%, and my chances at a far more respectable 63.2%.

When we run this as a class investigation, our experimental probability comes very close to this result after about 60 trials (including trials for homework!)

Needless to say, Jessica gets her 10p back when she learns that it was a scam, and gets the Mars Bar as well for being such a good sport.

Now is that why she keeps coming forward to volunteer?

OPTIONAL EXTRA

Apparently we only need the first and last letter of each word to be in the right place to enable us to read a piece of text and make sense of it. Provided the letters in between are all there "but not necessarily in the right order" then our brains can make good progress through the words. Worth investigating, anyway.

21. When 1/52 = 1 (One-Way Force Deck And Svengali Deck)

Effect
My Year 8s are just getting into Theoretical Probability so I like to lob this one in to keep them on their toes and get them thinking. Jeetander loves playing to the crowd, so I invite him up to help out. We do all the formal introductions, shaking hands and introducing ourselves. I ask him to confirm that we have never met before. (Why do they always agree??) Finally I ask him his age. He is thirteen. "That's incredible! What a coincidence! I was thirteen when I was your age too…"
Holding a fanned pack tightly towards me, I ask Jeetander to think of a card. Studying his expression as if reading the clues, I carefully select one card, hesitate, put it back then more confidently draw out a second one. "Now, Jeetander, what was your card?" He tells me that it was the six of hearts. "Oh well, it will work someday!" I shrug, replacing my card back in the pack.
[Brief discussion that the probability was 1/52 i.e. Very unlikely!!]
Far from being discouraged, I have a sudden change of plan. This time, I fan the cards face down towards Jeetander and ask him to select any card and show a few people, but NOT ME. This he does, pressing the card to his jacket in his efforts to conceal it. "Knowing you, Jeetander, you probably picked the three of diamonds."
Of course this is indeed his card. Calmly I return it to the deck, shuffling them openly towards the class until gradually they catch on that EVERY card in my pack is the three of diamonds!

Method
It is possible, but expensive, to make your own One-Way Force Deck as described above, since you will need to buy 52 identical packs of cards. Alternatively you can order them from any good magical supplier, of which there are increasingly many available online. [e.g. http://www.penguinmagic.com/europe/]
For the more advanced magician, and as a possible follow-up to baffle the students completely, it is possible to buy a special deck called the Svengali Deck. Since this is a commercial item I am not at liberty to divulge its secrets, but suffice it to say that you can present this trick as above but then knock everybody down flat by showing all the cards different at the end.
You can go one step further, with another special pack called the Brainwave Deck. In short, the spectator names any card, then you fan the deck out and show that her named card has magically reversed itself in the deck, being the only face up one there. That needs a great deal of practice, but it is so good I have frightened adults with it!

OTHER IDEAS
I like to create two little packs of ten cards, one with red backs and the other with blue backs. I take three cards from each pile and swap them over so that I have a "mainly blue" pile and a "mainly red" pile. Without showing the pupils which one I am holding, I place one out of sight under some books on the desk, and the other behind my back. Shuffling the cards a little bit (or pretending to!!) behind my back I bring the pack forward so that they can see the colour of the top card. I ask them which pile it is – mainly blue or mainly red. By controlling the cards I can make the pack display any colour of course, but eventually I do proper shuffles and the class gradually become more convinced by the evidence that the pack is the one they suspected.

22. Afghan Bands And The Squared Circle

<u>Effect</u>

"You remember that Circus that came to Valentines' Park last year?" I ask my bewildered Year 7s. Not that the question bewildered them – they are always bewildered. "Apparently there was a bit of drama behind the scenes." I go on to tell them about there being a highly specialised thief operating in the area, who took five belts from the performers' dressing rooms. Bobo and Bozo the Clowns, Mr Kawasaki the Sumo Wrestler and Fred and Ginger the Ballroom Dancers. The belts were never recovered, but as usual Mr Muscle the Strong Man saved the day.

"This is what he did" I say, holding up a piece of paper 6" by 30". "This is the Strong Man's belt which he cut up to help all the other out."

I cut it into three strips 2" wide and join their ends to make three loops of paper.

"First he needed belts for the two Clowns. That was easy!"

I cut round the loop and show the two belts.

"Next he needed a belt for the Sumo Wrestler. A bit harder that one!"

I cut round the second loop, and this time get one big belt.

"Finally the two ballroom dancers, who dance so closely they are like one person!"

I cut round the final loop and get two loops linked together.

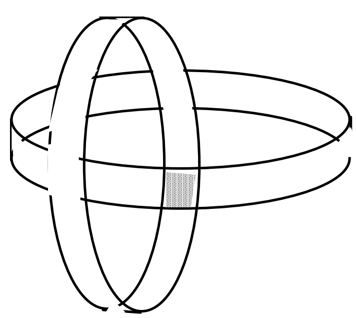
"Good old Mr Muscle – loves the jobs you hate. Or something like that."

<u>Method</u>

The Möbius Strip always gets a big "Wrah!" when first demonstrated, and children can't believe it happening even when they get to try it out for themselves. Obviously the three belts were made using (a) No twists (b) Half a twist (c) One full twist, and the trick then runs itself. Originally this story referred to Victorian Circus Performers such as the Fat Lady and the Siamese Twins, so this is an attempt at a slightly more PC version!

PLENARY

After the children have investigated the various loop cutting questions (e.g. what if you cut the Sumo Wrestler's belt around the middle? What if you make a half twist and cut one-third of the way along the width rather than half way?) there is another paper-cutting poser they may enjoy thinking about:



Join two ordinary circular loops back to back at right angles as shown in the terrible diagram on the left.

Cut around the middle of each one.

Did it surprise you as well???

23. Super Memory – The Journey Method and Pi

<u>Effect</u>

"Will you remember me in six months' time?" I ask my Year 10 class. They assure me that they will. "Will you remember me in a year's time?" Again they agree. "Will you remember me in five years' time?" Starting to get a bit wary, they agree, that yes, they probably will. "Knock! Knock!" I say. "Who's there?" they chorus together. "See? You've forgotten me already…"

I ask them for a list of ten really random objects. They come up with a bizarre assortment: Mark's trainers, an aeroplane, Chloe's chewing gum, a pencil etc. I write the list up on the board as they call them out and then ask them "What have they got in common?" I let them study the list for a minute. Then I cover the list up and ask them to write down the whole list from memory, in order!

Initially results are woeful, but when they have learned the journey method, they are soon able to learn even longer lists with ease. By the end of the lesson, the whole class can say the original list of ten objects either forwards or backwards, starting anywhere in the list.

<u>Method</u>

At last – a trick with obvious benefit! Children quickly grasp the usefulness of developing their memory when it comes to learning facts and figures from other subjects. The journey method, like all other efficient memory techniques, relies on the power of association, and particularly on visualisation.

Each student is asked to write down a ten stage journey, such as the journey to school. A stage can be any length, but it must be marked by a very clear landmark and have a definite order. It might begin: My bedroom, cleaning my teeth, my front gate, knocking at Gagan's house, the bus stop, the bus, … and so on.

The next step is to associate each item with the landmark for that part of the journey. The most effective way is to make the association as ridiculous as possible, involving many senses. For example, in MY BEDROOM are MARK'S ENORMOUS pair of SMELLY OLD TRAINERS, so big that they cover the bed. The smell is terrible so I run for the BATHROOM. Out of the window I see a RED and WHITE striped AEROPLANE, just like my TOOTHPASTE. It is so close that I can see everyone inside the Aeroplane CLEANING THEIR TEETH. When I get downstairs and outside I have REAL TROUBLE opening the FRONT GATE because CHLOE has COVERED IT with CHEWING GUM. And so the story continues. The more vivid each picture, the easier it is to recall.

<u>Pi Memorising</u>

There's nothing very magical about Pi really. Here's Pi to about 100 places: 3.14159265358979323846264338327950288419716939937510582097494459230781640628620899862803482534211706679 My "PB" is 120 places, but that has been beaten by pupils. I simply give them a sheet of the digits and leave them to it:

- 15 places = Hall of Fame
- 25 places = Bronze Certificate
- 50 places = Silver Certificate
- 100 places = Gold Certificate
- 150 places = Platinum Certificate

Other people think they are magic when they can recite them in class though!

24. Calendars and Hundred Squares

Effect

My Year 9s are getting pretty good with their Algebra now, so I think it is time for them to try this little challenge. I invite the new Greek boy, Giorgios, up to help. (“Sir, tell them! They keep calling me Gorgeous!”) On the board is a page from a calendar. I ask Giorgios to circle a square of 9 dates (Greeks know that you can’t square a circle…). Although I can’t see the board, this is what he has done:

				1	2	3
4	**5**	**6**	**7**	8	9	10
11	**12**	**13**	**14**	15	16	17
18	**19**	**20**	**21**	22	23	24
25	26	27	28	29	30	31

I ask Giorgios to tell me the total of the smallest and the largest numbers.
He says 26. Immediately I call out all nine dates in his square.
Giorgios changes places with Lauren, who chooses a different set of nine dates. This time I ask her for the centre date in her square. I then tell her the total of all the dates she selected! Lauren gives a little squeak of joy and skips back to her seat.
Next we turn our attention to the hundred square on the wall. This time I ask each pupil to choose a 4 x 4 square of numbers. I ask them to call out the total of their smallest and biggest and I then respond by telling them their first row.
They have to admit, they were a tiny bit impressed that time!

Method

We do a bit of mental algebra to start us off. If I am thinking of a number d, what is the number after mine? After discounting the dozens who are convinced it is e we finally agree that we can write it as $d + 1$. My number plus seven? My number plus ten? Plus twenty? Plus two then plus fourteen? Soon they are screaming out the answers as fast as I can make up the questions.
We draw three grids, one for each trick:

d	$d + 1$	$d + 2$
$d + 7$	$d + 8$	$d + 9$
$d + 14$	$d + 15$	$d + 16$

Giorgios tells me $2d + 16$. I then just divide by 2 and subtract 8 to get the first number, then count on.

$d - 8$	$d - 7$	$d - 6$
$d - 1$	d	$d + 1$
$d + 6$	$d + 7$	$d + 8$

The total is just $9d$ since all those + and – numbers cancel out. I just multiply Lauren’s number by 9.

d	$d + 1$	$d + 2$	$d + 3$
$d + 10$	$d + 11$	$d + 12$	$d + 13$
$d + 20$	$d + 21$	$d + 22$	$d + 23$
$d + 30$	$d + 31$	$d + 32$	$d + 33$

I subtract 30, then 3, from $2d + 33$, then halve the result. Once I know the first number, the first row is easy.

25. Total Disbelief

WARNING: This is easily the strongest effect in the whole collection. Do not be too ready to give away the method except for the benefit of your pupils!

Effect

As my Year 10s enter the classroom I hand them a 4 x 4 grid of seemingly random numbers. As they start to sit down, they begin comparing their grids with one another – they are all different! I have several spares, so I give these out as well. Pupils CHOOSE three numbers on the grid by ringing them, then they ring the one that is left. When they add up their four numbers, they are in for a shock. Despite the fact that all the grids were different, despite the fact that each pupil had a seemingly fair choice of numbers, EVERY TOTAL IN THE ROOM IS IDENTICAL. OK, they admit, that was scary! What is going on? We try it again and the result is the same, no matter what they do. It simply doesn't make sense!

Method

The grid is really 5 x 5 squares, but I "accidentally" chop off the top row and the left hand column before handing them out.

	7	**10**	**4**	**8**
6	13	16	10	14
11	18	21	15	19
9	16	19	13	17
5	12	15	9	13

Inside the grid I complete the addition square for the "seed numbers" which are randomly placed around the top and the side. The total of my seed numbers is:
$4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 = 60$ so this will be my predicted total.

The ringing procedure is as follows: Ring any number and cross out the others in the same row and the same column. Now ring one of the remaining 9 numbers and cross out the others in that row and column too. You will have four numbers left. Choose any one, then cross out the other one in its row and the other one in its column. Finally ring the last number. Your total will always be 60. (Try it!)

Explanation – make an addition grid for the letters *a, b, c, d, e, f, g, h* and go through the ringing procedure. You will notice that once you have chosen, say, $d + f$, you then have to cross out all the other *d*s and all the other *f*s. When you have finished your four rings contain between them the eight letters, so adding them up gives you the sum of the original 8 numbers.

Different squares are made possible by simply re-arranging the seed numbers around the edges of the grid. In fact, you don't even need to stick to exactly the same 8 numbers. Any 8 numbers which add up to 60 will give you the same result. Of course, you can alter the grid to force absolutely any total you wish.

Excel Notes for No. 25 Total Disbelief

I created an Excel spreadsheet which generates page after page of these squares to force the number 76. I used it to produce 120 different grids for an evening show on Mathemagic for parents and children taking part in Maths Week at Newbury Park Primary School. I did not reveal the method there, obviously, but offer it now for any of you who are called to present something similar in the future, say in an Assembly or with your own classes. I will happily email the spreadsheet to anyone who wishes it – just contact me at MrF@MathsIsFun.net with a suitable message. It does contain macros, however, so do not be too alarmed if your virus checker gets into a panic. It's quite safe!

Those of you with Excel experience will probably want to try making one up for yourself. This is true especially if you want to create grids with another total. You may want to take a collection of 40-forcing grids with you to help a friend celebrate their 40th birthday! Here then are some notes to help, to save having to solve all the programming problems from scratch.

Think about what you want to print off. I chose to print six grids on a sheet of A4, so I Set Print Area of my spreadsheet to just those six grids, and set the Page Setup to Portrait – Fit to 1 page. Only the grid cells needed cell borders, so I blanked out the rest of the sheet in the printed area.

These six grids are your addition grids. The seed numbers are around their edges so these have to be set to invisible font colour (usually white).

This is how I used random numbers to generate all those different grids. For each of my six grids in my table I set up a space on my spreadsheet for the following:

1) A list of eight random numbers [=rand()]
2) A RANK column which gives the position of each random number in a sorted list of all eight of them [=RANK(A22,A$22:A$29)]
3) A SCALE function which converts the RANK numbers (1 to 8 in some order) into my numbers (I used 6 to 13 to make 76, so my scale function was just =B22+5)

The cells along the top and left edges of my grid then contained copies of these last values. (=C22, =C23, =C24 etc.). I repeated all this for the other five grids.

The first macro I scripted was then:

1) Recalculate (makes new random numbers by pressing F9)
2) Print
3) Confirm

I assigned this to a button marked "Print new set"

The second macro was then

1) Click "Print new set"
2) (Click four more times)

I assigned this to a button marked "Print 5 new sets"

Since there were six grids to a page, I only needed to click this second button four times to create 120 different grids.

A1 Bonus – The French Drop

Effect

Kirsty asks to borrow a pencil sharpener. I approach her holding the pencil sharpener in my left hand. As she watches, I take it with my right hand – Kirsty can see that my left hand is now empty, but is alarmed to see that my right hand is empty too! All is not lost, however, for the pencil sharpener is then taken out of her pencil case which was on her desk all along.

Method

This is not easy, but more than repays the effort spent in getting it right. When done properly, it is a perfect combination of timing, misdirection and illusion.

I perform this by starting with the (coin / pencil sharpener / rubber etc.) held between my middle finger and left thumb, with palm uppermost. To be convincing, first get used to watching yourself in a mirror as you take the pencil sharpener out of your left hand with your right. Notice that your left hand fingers remain apart, and that your eyes follow the pencil sharpener as it moves away in the right hand. THIS MUST BE EXACTLY THE SAME AS WHEN YOU PERFORM THE FRENCH DROP.

The drop itself is not difficult – you just have to trust it has happened. Whatever you do, DON'T look at your left hand to check! You will ruin the illusion. As your right hand comes over the pencil sharpener, your thumb pulls back ever so slightly and stays there, allowing the pencil sharpener to fall. It is safely hidden by the wall of four fingers, but you MUST NOT HESITATE. Immediately close your right hand around the "Pencil sharpener" and watch it as you go to give it to your student. If they are following your right hand as well then you know it has worked. All this time your left hand has been open and apparently empty. Now you can drop it to your side but keep the pencil sharpener hidden in your relaxed palm. This is called "palming".

To produce the pencil sharpener from Kirsty's own pencil case I simply reach in with my apparently empty left hand, and using the cover of my fingers again, allow the pencil sharpener to drop down as far as my thumb and finger tips. As soon as it is in position I then simply left it up for her to see and place it on the desk.

There's a feeble magic trick well known to grandparents in which a coin held in the right hand is made to travel magically all the way along the arm, through the body and along the other arm to the closed left fist, then IMMEDIATELY back along its journey until it "arrives" back in the right hand again! Even small children complain about that one. "It just stays in your right hand!" they wail. I do the same trick, wait for the howls of protest, then do it again, but with a quick French drop at the start. This time, when I am half way through my story, they moan and complain like all the best teenagers. "Ha! Gotcha! Open your left hand! Prove it then!"

So I do.

I have had otherwise normal kids run screaming from the room.

A2 Bonus – Ruler Magic

Effect
I ask Jade to try this as she has a steady hand and is not easily fooled. “All you have to do,” I explain, “is keep one hand still and just move the other”.
I ask Jade to put out two fingers, her two index fingers, pointing towards me, so that I can balance the 30cm ruler on them. Jade has one finger under the zero and another under the 30cm mark. “Remember, just keep one hand still while you move the other. Now bring your hands together.”
Try as she might, the ruler will simply not sit still on either side, but slides first on one side then on the other until eventually, both fingers come to rest under the 15cm mark. She tries again, but can not prevent this from happening.
“Fair enough, I’ll change the rules just for you. Now both hands MUST move. Just slide your fingers back to the starting point.”
Poor old Jade cannot succeed this time either. One finger acts as if it is welded to the ruler and the other happily slides to the end.

Method
This works just as described, and is a constant source of wonder to pupils, all of whom think that they will be the first to break the laws of friction and gravity. In the first case, both fingers act as pivots. In the second, one finger fixes itself as a pivot and the other has no supporting role.

A3 Bonus – Dice Magic

Effect
“Let me see that one, Barry. Hmm. 4 on one side and 2 on the other? That’s not right is it? I thought it was meant to be a three on the other side. Let me give it a quick rub. Now let’s see. Ah! That’s better – 4 on one side and 3 on the other. You can use it now…”

Method
This is most effective with small dice until you get used to the action. As with the French Drop, you must first practise in front of a mirror turning over the die normally. I hold it in my right hand, thumb on the bottom and fingers on top. When I turn my wrist to show the other side, my thumb becomes the top and the fingers are below. On the way round, however, I execute the famous Paddle Move. This is an imperceptible roll of 1/4 turn which I always do by moving my thumb to reveal the “wrong” opposite number. Doing this in reverse, I come back to the 4, at which point I rub the “two” (actually the three), then turn it over again without the Paddle Move. Now the three is in view as expected, so I drop the die on the desk.
You can guarantee that every child will want to pick that die up and try rubbing the spots on and off!

Of course I would never actually give a die back as simply as that. I would always French Drop it, just to give them something else to puzzle over…

A4 Bonus – Finger Magic

Flexible Fingers
Palms together. Slide your right hand up enough for you to be able to curl your right fingers over your left hand. Immediately pull it down and raise your left hand until your left fingers can curl over your right hand. Repeat this in quick succession to give the illusion that your fingers are bending backwards as if made of rubber.

Stretching Finger
Make a pointing finger with your right hand, i.e. all fingers except the index finger balled into a fist. Make a V between the first and second fingers of your palm-down left hand. Into this V from below, slowly slide in your right index finger, until it is the same length as your left index finger. THEN KEEP GOING until your right finger looks to be at least one and a half times as long as the other one. This is most effective if you can have your right index finger "stretched" by somebody holding the tip, so that you can give all the anguished sound effects which children naturally find terribly amusing.

Crazy Finger Counting
Count steadily from one to six as you tap along your fingers, but although you start and finish in sync, you cunningly make the counting slightly faster than the tapping, so that you pronounce the word "six" just as you tap your last finger.
When you can do this evenly, you will even fool yourself!
Miscounting objects like this will enable you to make things appear and disappear at will. That pile of Multi-Link: you count "ten" cubes, then magically "take one away" – Oh look! Now there's nine!

Thumb Removal (1)
Bend both thumbs and fit them together knuckle to knuckle so that it looks like one whole thumb. Cover the join with your index finger. Now approach a sensitive child and say you think you've had an accident. Lift the top part of your thumb away to show the extent of the damage.

Thumb Removal (2)
Left hand, thumb up, hand open, palm towards your body. Right hand makes a fist around the left thumb and slides it along the left hand. I usually take it right away, inspect it, then replace it. Wiggle your thumb as if recovering from surgery.
As the right hand closes around the left thumb, tuck the left thumb down well below the line of the top of the left hand. You can actually bend your thumb down out of sight AFTER it has been gripped by the right fist as this strengthens the illusion.

Surprising Finger Jump
To illustrate 1 + 1 = 2, you proceed as follows. Make two pointing fingers, one on each hand, with fingers pointing up and palms facing forwards. Say "One plus one makes…" and bring your hands quickly towards each other so that the fists bash together and the fingers touch. Shake your head. Repeat. On the third time as your hands come together, bend your right index finger down and simultaneously raise your left middle finger, so that it looks like one finger has jumped across to the other hand. "…two!"